Révision du Bloc 1

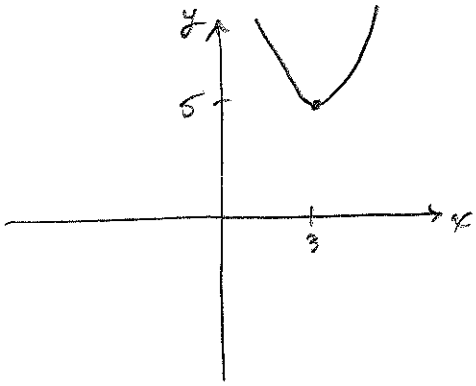
(Corrigé)

RAS 3.1

#1. a) Une altitude de 0 km à 5 km $x \in [0,5] \cup [40,55] \cup [95,\infty[$
de 40 km à 55 km
de 95 km à ∞.

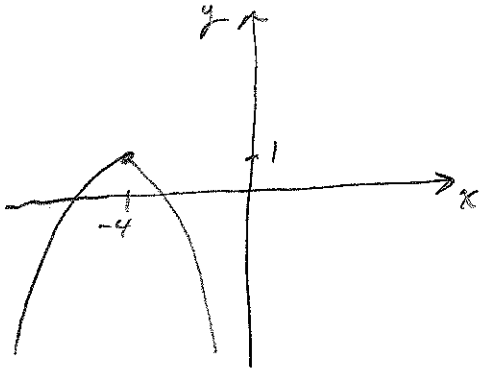
b) Croissance : $x \in [10,50] \cup [80,\infty[$
décroissance : $x \in [0,25] \cup [50,80]$

#2. a)



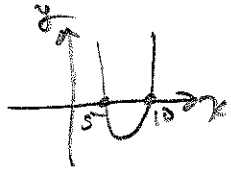
Croissant : $x \in [3,\infty[$
décroissant : $x \in]-\infty,3]$

b)



Croissant : $x \in]-\infty,-4]$
décroissant : $x \in [-4,\infty[$

#3. a) $f(x) = x^2 - 15x + 50$ → a⊕ ⇒

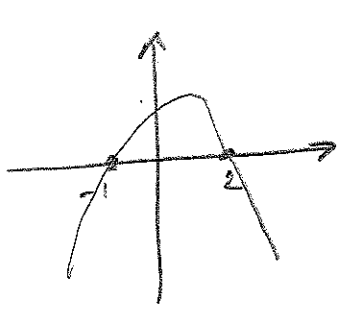


$0 = x^2 - 15x + 50$ $\quad -5 \times -10 = 50$
$0 = x^2 - 5x - 10x + 50$ $\quad -5 + -10 = -15$
$0 = x(x-5) - 10(x-5)$
$0 = (x-5)(x-10)$

$x - 5 = 0$; $x - 10 = 0$
$x = 5$; $x = 10$

signe + : $x \in]-\infty,5] \cup [10,\infty[$
signe − : $x \in [5,10]$

b) $g(x) = -4x^2 + 4x + 8$ (a < 0) $\Rightarrow$



$\frac{0}{-4} = \frac{-4x^2 + 4x + 8}{-4}$

$0 = x^2 - x - 2$ $\quad -2 \times 1 = -2$, $-2 + 1 = -1$

$0 = x^2 - 2x + x - 2$

$0 = x(x-2) + 1(x-2)$

$0 = (x-2)(x+1)$

$x = 2$ $\quad x = -1$

signe +: $x \in [-1, 2]$

signe −: $x \in]-\infty, -1] \cup [2, \infty[$

RAS 3.2

#1. a) $y = a(x-h)^2 + k$

$25 = a(14-7)^2 + 30$

$25 = a(7)^2 + 30$

$\frac{-5}{49} = \frac{49a}{49}$ $\Rightarrow$ $y = \frac{-5}{49}(x-7)^2 + 30$

$a = \frac{-5}{49}$

b) $\approx$ 24 secondes ou plus précis $\Rightarrow$ 24,15 sec. (remplacer y par 0)

c) $28 = \frac{-5}{49}(x-7)^2 + 30$

$-2 = \frac{-5}{49}(x-7)^2$

$\pm\sqrt{19,6} = \sqrt{(x-7)^2}$

$\pm 4,43 = x - 7$ $\Rightarrow$ $x = 7 \pm 4,43$

$x = 11,43$ ou $x = 2,57$

$11,43 - 2,57 = 8,86$

Pendant 8,86 sec.

#2.

a) $f(x) = 3x^2 + 18x + 24$ $\quad (6/2)^2 = 9$

$f(x) = 3(x^2 + 6x + 9 - 9) + 24$

$f(x) = 3(x+3)^2 + 24 - 27$

$\boxed{f(x) = 3(x+3)^2 - 3}$ → F. Canonique

$f(x) = 3x^2 + 18x + 24$

$f(x) = 3(x^2 + 6x + 8)$ $\quad 2 \times 4 = 8$, $2 + 4 = 6$

$f(x) = 3[x^2 + 2x + 4x + 8]$

$f(x) = 3[x(x+2) + 4(x+2)]$

$\boxed{f(x) = 3(x+2)(x+4)}$ → F. Factorisée

b) $g(x) = -4(x+3)^2 + 40$

$g(x) = -4(x+3)(x+3) + 40$

$g(x) = -4(x^2 + 6x + 9) + 40$

$g(x) = -4x^2 - 24x - 36 + 40$

$\boxed{g(x) = -4x^2 - 24x + 4}$ → F. Générale

$\boxed{g(x) = -4(x^2 + 6x - 1)}$

→ F. Factorisée

$x + x = -1$, $x + x = 6$

c) $h(x) = 0{,}25(x-5)(x+1)$

$h(x) = \frac{1}{4}(x^2 - 5x + x - 5)$

$\boxed{h(x) = \frac{1}{4}x^2 - x - \frac{5}{4}}$ → F. générale

$h(x) = \frac{1}{4}x^2 - x - \frac{5}{4}$

$h(x) = \frac{1}{4}(x^2 - 4x \underline{+4-4}) - \frac{5}{4}$ $\quad \left(\frac{4}{2}\right)^2 = 4$

$h(x) = \frac{1}{4}(x-2)^2 - \frac{5}{4} - \frac{4}{4}$

$\boxed{h(x) = \frac{1}{4}(x-2)^2 - \frac{9}{4}}$ → F. Canonique

d) $i(x) = \frac{2}{3}(x-6)(x-6) + 30$

$i(x) = \frac{2}{3}(x^2 - 12x + 36) + 30$

$i(x) = \frac{2}{3}x^2 - 8x + 24 + 30$

$\boxed{i(x) = \frac{2}{3}x^2 - 8x + 54}$ → F. générale

e) $j(x) = -10\left(x + \frac{1}{2}\right)(x-7)$

$j(x) = -10\left(x^2 - 7x + \frac{1}{2}x - \frac{7}{2}\right)$

$j(x) = -10x^2 + 70x - 5x + 35$

$\boxed{j(x) = -10x^2 + 65x + 35}$ → F. générale

$\left(\frac{13}{2} \times \frac{1}{2}\right)^2 = \frac{169}{16}$

$j(x) = -10\left(x^2 - \frac{13}{2}x + \frac{169}{16} - \frac{169}{16}\right) + 35$

$j(x) = -10\left(x - \frac{13}{4}\right)^2 + 35 + \frac{1690}{16}$

$j(x) = -10\left(x - \frac{13}{4}\right)^2 + 140{,}625$

ou

$\boxed{j(x) = -10\left(x - \frac{13}{4}\right)^2 + 140{,}625}$ F. Canonique

f) $K(x) = -2x^2 - 8x - 15$

$K(x) = -2\left(x^2 + 4x + 4 - 4\right) - 15$

$K(x) = -2(x+2)^2 - 15 + 8$

$\boxed{K(x) = -2(x+2)^2 - 7}$ → F. Canonique

R.AS 2.3

#1. $I = ?$

$C = 5000\$$

3 ans

$i = \frac{6,8\%}{2}$

$n = 3 \times 2 = 6$

$M = C + I$

$M = C(1+i)^n$

$M = 5000\left(1 + \frac{0,068}{2}\right)^6$

$M = 6110,73\ \$$

$I = M - C$

$I = 6110,73 - 5000$

$\boxed{I = 1110,73\ \$}$

#2. 4 ans

$M = 4550\$$

$C = ?$

$i = \frac{4,25\%}{12}$

$n = 4 \times 12 = 48$

$M = C(1+i)^n$

$4550 = C\left(1 + \frac{0,0425}{12}\right)^{48}$

$C = \frac{4550}{\left(1 + \frac{0,0425}{12}\right)^{48}}$

$\boxed{C = 3839,83\ \$}$

#3. $+4\%$

$n = 5$ ans

$\frac{27}{\text{5 ans passés}}$, $\frac{\quad}{\text{4 ans passés}}$, $\frac{\quad}{\text{3 ans "}}$, $\frac{\quad}{\text{2 ans "}}$, $\frac{\quad}{\text{1 an "}}$, $\frac{?}{\quad}$

$C = 27$

$M = C(1+i)^n$

$M = 27(1+0{,}04)^5$

$M = 32{,}85 \approx$ **33 accrochages**

#4. $+3{,}5\%$

$M = 28000\$$

$n = 10$ ans

$M = C(1+i)^n$

$28000 = C(1+0{,}035)^{10}$

$C = \frac{28000}{(1+0{,}035)^{10}}$

$C = 19\,849{,}73\$$

#5. -18%

$M = 1380\$$

$n = 6$ ans

$M = C(1+i)^n$

$1380 = C(1-0{,}18)^6$

$C = 4539{,}37\$$

RAS 3.5

#1. a) $x + 2y = 15 \rightarrow x = 15 - 2y$

$4x - y = 6$

$4(15-2y) - y = 6$

$60 - 8y - y = 6$

$-9y = -54$

$\frac{-9y}{-9} = \frac{-54}{-9}$

$y = 6$

$x = 15 - 2(6)$

$x = 3$

b) $4x + y = -10 \rightarrow y = -4x - 10$

$3y + 9 = x$

$3(-4x-10) + 9 = x$

$-12x - 30 + 9 = x$

$-13x = 21$

$x = \frac{-21}{13}$

$y = -4\left(\frac{-21}{13}\right) - 10$

$y = \frac{84}{13} - 10$

$y = \frac{84}{13} - \frac{130}{13}$

$y = \frac{-46}{13}$

#2. x : $ c. couleur

y : $ c. noir & blanc

$x = 3y + 0{,}03$

$8x + 6y = 1{,}38$

#3.

x : # bouteille (4L)

y : # bouteille (500mL $\Leftrightarrow$ 1/2 L)

$x + y = 270 \rightarrow y = 270 - x$

$4x + \frac{1}{2}y = 275$

$4x + \frac{1}{2}(270 - x) = 275$

$4x + 135 - 0{,}5x = 275$

$3{,}5x = 140$

$x = 40$

$y = 270 - 40$

$y = 230$

40 bouteilles de 4L
et
230 bouteilles de 500mL

#4. a) x : # de boîtes d'oranges (14$) $\xrightarrow{-8\$}$ Profit 6$/boîtes

y : # de boîtes de pamplemousses (12$) $\xrightarrow{-7}$ Profit 5$/boîtes

b) $16x + 5y \leq 20\,000$

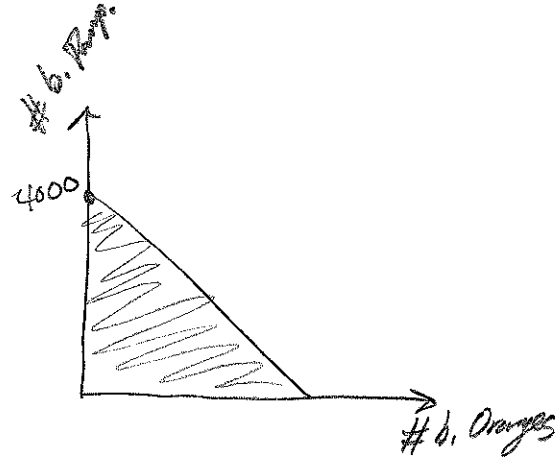
rest. $\rightarrow x \geq 0$

$y \geq 0$

c) $5y \leq -6x + 20\,000$

$y \leq -\frac{6}{5}x + \frac{20\,000}{5}$

$y \leq -\frac{6}{5}x + 4000$



#5.

a) $x \geq 0$

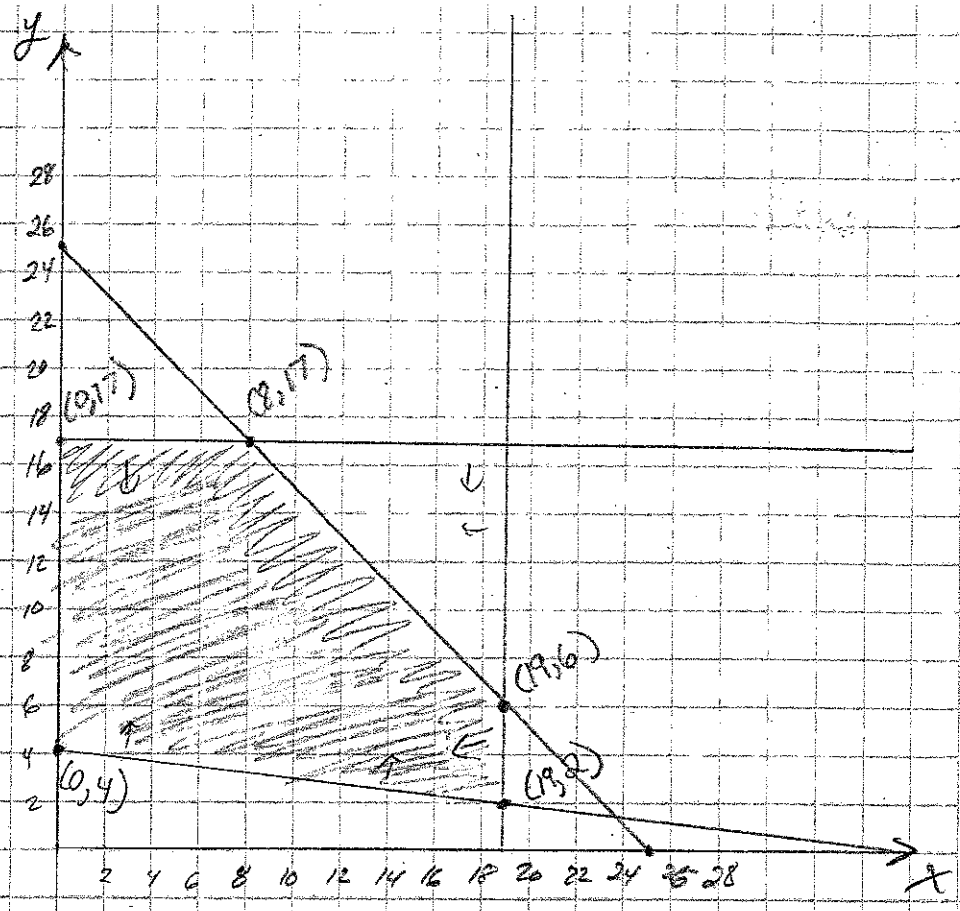
$y \geq 0$

$x \leq 19$

$y \leq 17$

$$\begin{cases} 2x + 19y - 76 \geq 0 \\ \frac{19y}{19} \geq \frac{-2x}{19} + \frac{76}{19} \\ y \geq \frac{-2}{19}x + 4 \end{cases}$$

$y \leq -x + 25$



b) $x \geq 0$

$y \geq 0$

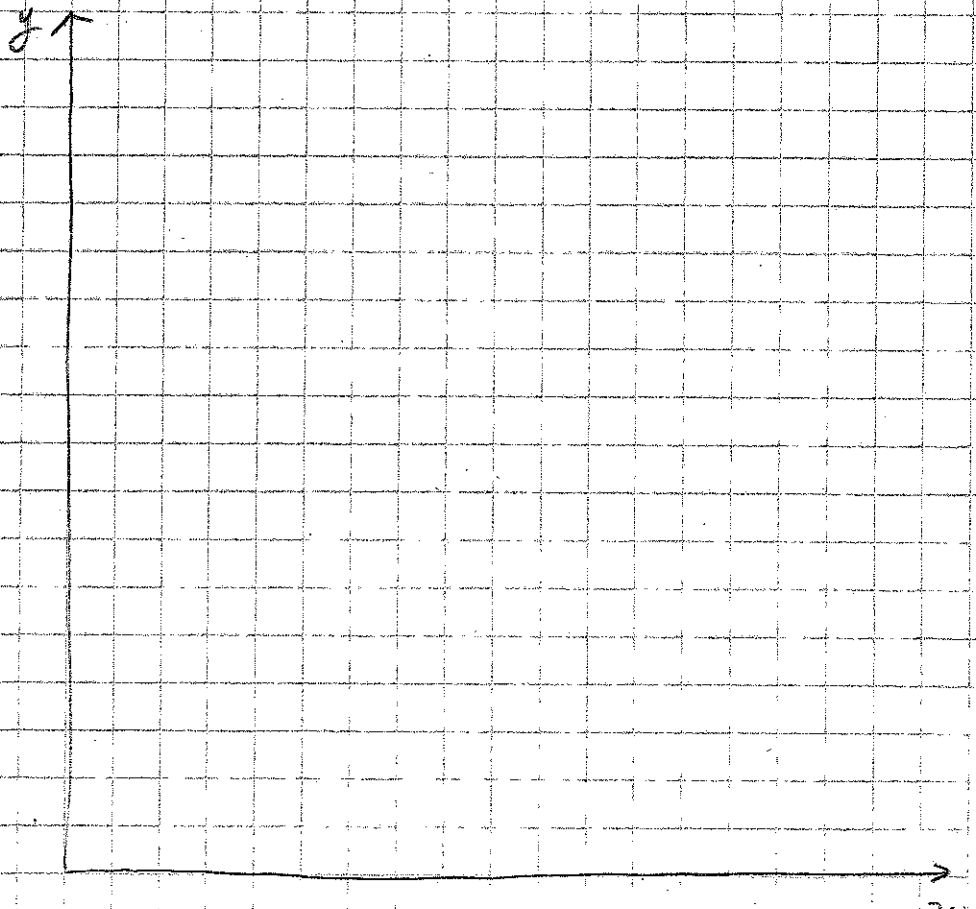
$$\begin{cases} -23y \geq -18x - 304 \\ y \leq \frac{18}{23}x + \frac{304}{23} \end{cases}$$

$$\begin{cases} 3y \leq -x + 120 \\ y \leq \frac{-1}{3}x + 40 \end{cases}$$

$$\begin{cases} -6y \leq -5x + 54 \\ y \leq \frac{5}{6}x - 9 \end{cases}$$

$-13y \leq -x - 178$

$y \geq \frac{1}{13}x + \frac{178}{13}$



#6. x : # billets de 5$
y : # billets de 10$
z : # billets de 20$

① $x + y + z = 71$

② $5x + 10y + 20z = 925$

③ $z = x + y - 7 \rightarrow x + y - z = 7$

$5 \times$ ① + ②

$-5x - 5y - 5z = -355$
$+ \quad 5x + 10y + 20z = 925$

④ $5y + 15z = 570$

$5y + 15(32) = 570$
$5y + 480 = 570$
$5y = 90$
$\boxed{y = 18}$

$-1 \times$ ① + ③

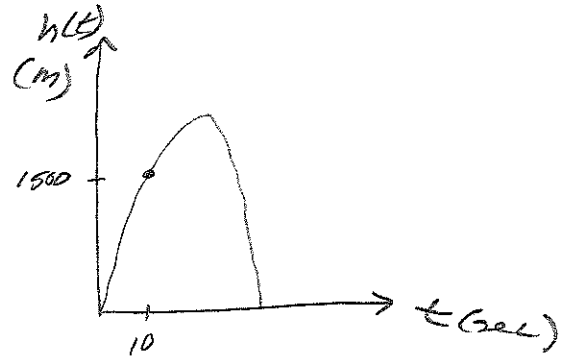
$-x - y - z = -71$
$+ \quad x + y - z = 7$

$-2z = -64$
$\frac{-2z}{-2} = \frac{-64}{-2}$
$\boxed{z = 32}$

① $x + y + z = 71$
$x + 18 + 32 = 71$
$\boxed{x = 21}$

Il y a 21 billets de 5$
18 billets de 10$
32 billets de 20$

#7. $h = at^2 + bt + c$



$(10, 1500)$ $(20, 2000)$ $(30, 1500)$

① $(10, 1500)$
$x \quad y$

$1500 = a(10)^2 + b(10) + c$

$\boxed{1500 = 100a + 10b + c}$

② $(20, 2000)$
$x \quad y$

$2000 = a(20)^2 + b(20) + c$

$\boxed{2000 = 400a + 20b + c}$

③ $(30, 1500)$
$x \quad y$

$1500 = a(30)^2 + b(30) + c$

$\boxed{1500 = 900a + 30b + c}$

① & ② × -1

$100a + 10b + c = 1500$
$+ \; -400a - 20b - c = -2000$
④ $-300a - 10b = -500$

① & ③ × -1

$100a + 10b + c = 1500$
$+ \; -900a - 30b - c = -1500$
⑤ $-800a - 20b = 0$

-2 × ④ + ⑤

$600a + 20b = 1000$
$+ \; -800a - 20b = 0$
$-200a = 1000$

$\boxed{a = -5}$

remplace dans ⑤

$-800(-5) - 20b = 0$
$-20b = -4000$

$\boxed{b = 200}$

remplace dans ①

$1500 = 100a + 10b + c$
$1500 = 100(-5) + 10(200) + c$

$\boxed{c = 0}$

RHS 3.8/3.6

#1. a) $6x^2 - 15x + 4 = 21$

$6x^2 - 15x - 17 = 0$

$-x \times = -102$

$-+ \times = -15$

$\frac{6}{1}\left(x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16}\right) - 17 = 0 \quad \left(\frac{5}{2} \times \frac{1}{2}\right)^2 = \frac{25}{16}$

$6\left(x - \frac{5}{4}\right)^2 - 17 - \frac{6(25)}{16} = 0$

$6\left(x - \frac{5}{4}\right)^2 = +17 + \frac{6(25)}{16}$

$6\left(x - \frac{5}{4}\right)^2 = \frac{272}{16} + \frac{150}{16}$

$\frac{1}{6} \times 6\left(x - \frac{5}{4}\right)^2 = \frac{422}{16} \times \frac{1}{6}$

$\sqrt{\left(x - \frac{5}{4}\right)^2} = \sqrt{\frac{211}{48}}$ $\longrightarrow \sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$

$x - \frac{5}{4} = \pm \frac{\sqrt{211}}{4\sqrt{3}}$

$x = \frac{5}{4} + \frac{\sqrt{211}}{4\sqrt{3}}$ et $x = \frac{5}{4} - \frac{\sqrt{211}}{4\sqrt{3}}$

$x = \frac{5}{4} + \frac{\sqrt{633}}{12}$ et $x = \frac{5}{4} - \frac{\sqrt{633}}{12}$

$x = \frac{15 + \sqrt{633}}{12}$ et $x = \frac{15 - \sqrt{633}}{12}$

$x = 3{,}35$ et $x = -0{,}85$

#1 b) $x^2 - 6x - 3 = 0$

$-x \times = -3$

$- + - = -6$

$(x^2 - 6x + 9 - 9) - 3 = 0$ $\quad \left(\frac{6}{2}\right)^2$

$\sqrt{(x-3)^2} = \sqrt{12}$

$x - 3 = \pm\sqrt{12}$ $\quad \sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$

$x = 3 \pm \sqrt{12}$

$x = 3 \pm 2\sqrt{3}$

$x = 3 + 2\sqrt{3}$ or $x = 3 - 2\sqrt{3}$

c) $2(-2x^2 - x) = x^2 - 3$

$-4x^2 - 2x = x^2 - 3$

$-5x^2 - 2x + 3 = 0$ $\quad -5 \times 3 = -15$, $-5 + 3 = -2$

$-5x^2 - 5x + 3x + 3 = 0$

$-5x(x+1) + 3(x+1) = 0$

$(x+1)(-5x+3) = 0$

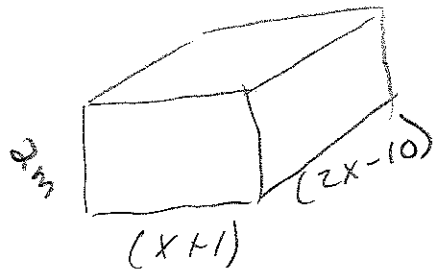
$x + 1 = 0$ or $-5x + 3 = 0$

$x = -1$

$-5x = -3$

$x = 3/5$

#2.



$V = 20\,m^3$

$$\frac{20}{2} = \frac{2(x+1)(2x-10)}{2}$$

$$10 = (x+1)(2x-10)$$

$$10 = 2x^2 - 10x + 2x - 10$$

$$0 = 2x^2 - 8x - 20$$

$$\frac{0}{2} = \frac{2(x^2-4x-10)}{2}$$

$$0 = x^2 - 4x - 10$$

$$x^2 - 4x - 10 = 0$$

$$(x^2 - 4x + 4 - 4) - 10 = 0 \qquad \left(\frac{4}{2}\right)^2$$

$$\sqrt{(x-2)^2} = \sqrt{14}$$

$$x - 2 = \pm\sqrt{14}$$

$$x = 2 \pm \sqrt{14}$$

$x = 2 + \sqrt{14}$ et ~~$x = 2 - \sqrt{14}$~~

$\boxed{x = 5,74}$ et ~~$x = -1,74$~~

#3. a) $x^2 + 4x - 21$

$1 \times -3 = -21$

$1 + -3 = 4$

$x^2 + 7x - 3x - 21$

$x(x+7) - 3(x+7)$

$\boxed{(x+7)(x-3)}$

b) $x^{16} - x^4$

$x^4(x^{12} - 1)$

$x^4(x^6+1)(x^6-1)$

$\boxed{x^4(x^6+1)(x^3+1)(x^3-1)}$

#4.

Aire $= 130\pi\ cm^2$

$A = 2\pi r^2 + 2\pi r h$

$$\frac{130\pi}{\pi} = \frac{2\pi r^2}{\pi} + \frac{2\pi r(8)}{\pi}$$

$$130 = 2r^2 + 16r$$

$$\frac{0}{2} = \frac{2r^2 + 16r - 130}{2}$$

$$0 = r^2 + 8r - 65$$

$$0 = r^2 + 8r - 65$$

$13 \times -5 = -65$

$13 + -5 = 8$

$$0 = r^2 + 13r - 5r - 65$$

$$0 = r(r+13) - 5(r+13)$$

$$0 = (r+13)(r-5)$$

$r + 13 = 0$

$r = \cancel{-13}$

$r - 5 = 0$

$\boxed{r = 5}$

RAS 4.1

#1. a) $15^2 = 8(2x+8)$ (Point ext. + tg + sec.)

$225 = 16x + 64$

$\frac{161}{16} = \frac{16x}{16}$

$x = 10,1$ cm

b) $x(2x) = 2(5)$ (Point. ext. + 2 sec.)

$\frac{2x^2}{2} = \frac{10}{2}$

$\sqrt{x^2} = \sqrt{5}$

$x = \pm\sqrt{5} \rightarrow x = +\sqrt{5}$ cm

c) $x = \frac{98}{2} = 49°$ (Angle inscrit + Arc)

$y = 98°$ (Angle au centre + Arc)

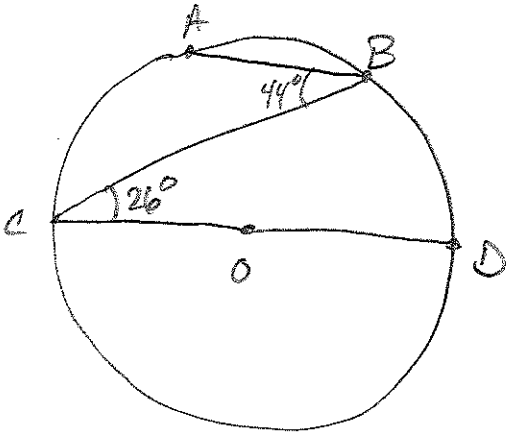
d) $\angle CEA = 180° - 38°$ (Angle Plat)

$\angle CEA = 142°$

$x = 180° - 142° - 22°$ (Somme des Angles dans un $\triangle$)

$x = 16°$

#2.



$\overline{OD}$ → rayon

$\overline{CD}$ → Diamètre

$\angle COD = \overset{\frown}{CD} = 180°$ (Angle au centre + Arc)

$\overset{\frown}{AC} = \angle ABC \times 2$ (Angle inscrit + Arc)

$\overset{\frown}{AC} = 44 \times 2$

$\overset{\frown}{AC} = 88°$

$\overset{\frown}{AB} = \overset{\frown}{CD} - \overset{\frown}{AC} - \overset{\frown}{BD}$

$\overset{\frown}{AB} = 180° - 88° - 52°$

$\boxed{\overset{\frown}{AB} = 40°}$

$\overset{\frown}{BD} = \angle BCD \times 2$ (Angle inscrit + Arc)

$\overset{\frown}{BD} = 26 \times 2$

$\overset{\frown}{BD} = 52°$

#3. $x(x+6,8) = 3(y+3)$ (Point ext. + 2 sec.)

$6,4x = 3y$ (2 cordes se croisent)

$x^2 + 6,8x = 3y + 9$

$y = \frac{6,4x}{3}$

$x^2 + 6,8x = 3\left(\frac{6,4x}{3}\right) + 9$

$x^2 + 6,8x = 6,4x + 9$

$x^2 + 0,4x - 9 = 0$

$x^2 + \frac{2}{5}x - 9 = 0$

$\left(x^2 + \frac{2}{5}x + \frac{1}{25} - \frac{1}{25}\right) - 9 = 0$

$\left(\frac{2}{5} \times \frac{1}{2}\right)^2 = \frac{1}{25}$

$\left(x + \frac{1}{5}\right)^2 - 9 - \frac{1}{25} = 0$

$\sqrt{\left(x + \frac{1}{5}\right)^2} = \sqrt{\frac{226}{25}}$

$x + \frac{1}{5} = \pm \frac{\sqrt{226}}{5}$

$x = -\frac{1}{5} \pm \frac{\sqrt{226}}{5}$

$\boxed{x = -\frac{1}{5} + \frac{\sqrt{226}}{5} = 2,81 \text{ cm}}$

$y = \frac{6,4(2,81)}{3}$

$\boxed{y = 6}$

#4. $\overline{AF} = \overline{DF}$ (Diamètre ⊥ corde)

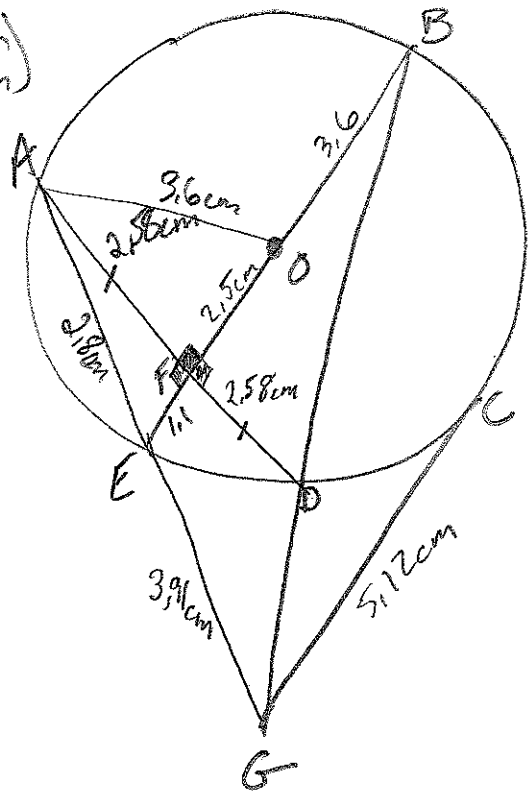
$\overline{AF} = 2{,}58\text{ cm}$

$\overline{AO}$ = rayon = $\frac{7{,}2}{2} = 3{,}6$ (définition rayon)

$3{,}6^2 = 2{,}58^2 + \overline{OF}^2$ (Th. Pythagore)

$3{,}6^2 - 2{,}58^2 = \overline{OF}^2$

$\overline{OF} = 2{,}5\text{ cm}$



$\overline{EF} = \overline{OE} - \overline{OF}$

$\overline{EF} = 3{,}6 - 2{,}5$ (rayon)

$\overline{EF} = 1{,}1$

$\overline{AE}^2 = 2{,}58^2 + 1{,}1^2$ (Th. Pythagore)

$\overline{AE} = 2{,}8\text{ cm}$

$\overline{CG}^2 = 3{,}91 \times (3{,}91 + 2{,}8)$ (Point ext. + tg + sec)

$\overline{CG} = 5{,}12\text{ cm}$

$\overline{BD}^2 = \overline{FD}^2 + \overline{FB}^2$ (Th. Pythagore)

$\overline{BD}^2$

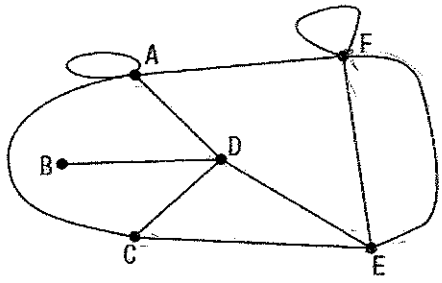
RAS 4.4 (Parcours B - seulement)

#1.

Pour chacun des graphes, déterminez :

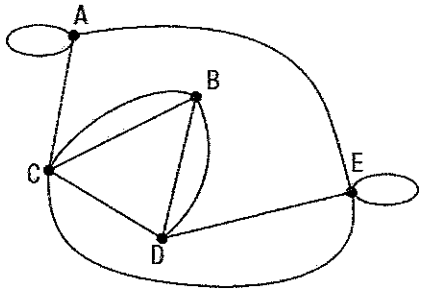
1) les sommets reliés par des arêtes parallèles;
2) l'ordre du graphe;
3) le degré de chaque sommet;
4) les sommets adjacents au sommet C;
5) les sommets qui ont une boucle.

a)



1) E et F
2) 6
3) A(4), B(1), C(3), D(4), E(4), F(4)
4) E, D, A
5) A, F

b)



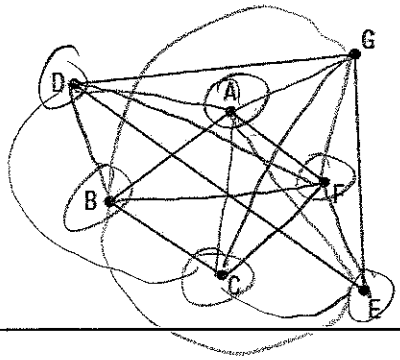
1) C et B ; B et D
2) 5
3) A(4), B(4), C(5), D(4), E(4)
4) B, D, A, E
5) A, E

#2.

Pour chacun des graphes :

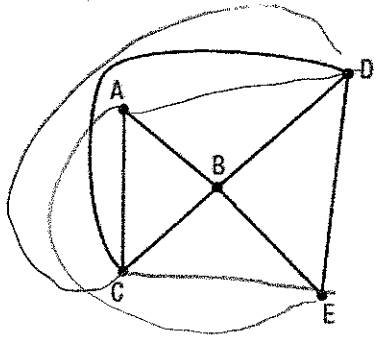
1) déterminez si le graphe est connexe;
2) déterminez le nombre d'arêtes qu'il faut ajouter au graphe pour qu'il soit complet.

a)



1) Non
2) 5+4+3+1+2 = (15)

b)



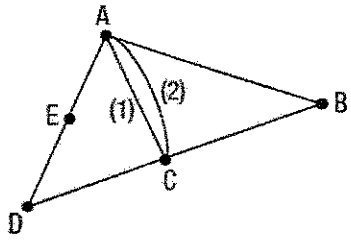
1) Oui
2) 2+1 = (3)

#3.

Pour chacun des graphes, nommez, si possible :

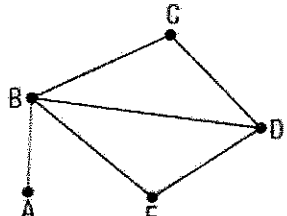
1) un cycle simple;
2) une chaîne eulérienne ou un cycle eulérien;
3) une chaîne hamiltonienne ou un cycle hamiltonien.

a)



1) A-B-C-A (Plusieurs réponses possibles)
2) A-E-D-C-A-C-B-A (PRP)
3) A-E-D-C-B-A (PRP)

b)



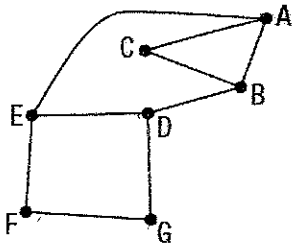
1) B-C-D-B (PRP)
2) A-B-E-D-B-C-D (PRP)
3) A-B-E-D-C (PRP)

#4.

Pour chacun des graphes :

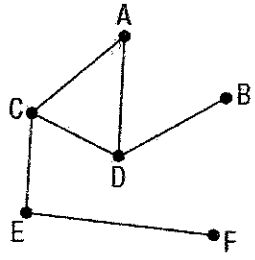
1) déterminez d(A, F);
2) nommez un cycle simple de longueur 3;
3) nommez une chaîne de longueur 6;
4) nommez la chaîne simple la plus longue qu'il est possible de former.

a)



1) 2
2) A-B-D-G (Plusieurs réponses possibles)
3) A-C-B-D-G-F-E (PRP)
4) A-C-B-D-G-F-E-D

b)



1) 3
2) A-C-E-F (Plusieurs réponses possibles)
3) B-D-C-A-D-C-E (PRP)
4) B-D-C-E-F (PRP)