

3.8 Les équations exponentielles

Corrigé Omnimaths 12^o

p. 90
20.

$$\frac{1}{64} = 1 \left(\frac{1}{2}\right)^{30/d}$$
$$\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{30/d}$$
$$6 = \frac{30}{d}$$
$$d = 5$$

21.

$$0,25 = 100 \left(\frac{1}{2}\right)^{40,8/d}$$
$$0,25 = \left(\frac{1}{2}\right)^{40,8/d}$$
$$\frac{1}{4} = \left(\frac{1}{2}\right)^{40,8/d}$$
$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{40,8/d}$$
$$2 = \frac{40,8}{d}$$
$$d = 20,4$$

22

$$\frac{1}{\sqrt{2}} = 1 \left(\frac{1}{2}\right)^{8/d}$$
$$\left(\frac{1}{2}\right)^{1/2} = \left(\frac{1}{2}\right)^{8/d}$$
$$\frac{1}{2} = \frac{8}{d}$$
$$d = 16$$

23

$$6,25 = 100 \left(\frac{1}{2}\right)^{1/d}$$
$$0,0625 = \left(\frac{1}{2}\right)^{1/d}$$
$$\frac{1}{16} = \left(\frac{1}{2}\right)^{1/d}$$
$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{1/d}$$
$$4 = \frac{1}{d}$$
$$d = \frac{1}{4} \text{ minute}$$

ou 30 secondes

25.

$$\frac{80000}{1250} = \frac{1250}{1250} (2)^{1,5/d}$$
$$64 = 2^{1,5/d}$$
$$2^6 = 2^{1,5/d}$$
$$6 = \frac{1,5}{d}$$
$$d = 0,25 \text{ h.}$$

ou 15 minutes

$$26 \text{ a) } Q = 40 \left(\frac{1}{2}\right)^{t/14.9}$$

$$Q = 4,29 \text{ g}$$

$$b) \frac{2.5}{40} = \frac{40 \left(\frac{1}{2}\right)^{t/14.9}}{40}$$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{t/14.9}$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{t/14.9}$$

$$4 = \frac{t}{14.9}$$

$$t = 59,6$$

$$27. \text{ a) } Q(t) = Q_0 \left(\frac{1}{2}\right)^{t/5.3}$$

$$Q(10.6) = Q_0 \left(\frac{1}{2}\right)^{10.6/5.3}$$

$$Q(10.6) = \frac{1}{4} Q_0$$

Il restera $\frac{1}{4}$ du cobalt 60

$$b) \frac{0,125 Q_0}{Q_0} = \frac{Q_0 \left(\frac{1}{2}\right)^{t/5.3}}{Q_0}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{t/5.3}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/5.3}$$

$$t = 5.3 \times 3$$

$$t = 15,9$$

$$28. \text{ a) } P = P_0 (4)^{t/7}$$

$$b) P = P_0 (4)^{2/7}$$

$$P = P_0 4^3$$

$$P = 64 P_0$$

$$c) \frac{2 P_0}{P_0} = \frac{P_0 4^{t/7}}{P_0}$$

$$2 = 4^{t/7}$$

$$2 = 2^{2t/7}$$

$$\frac{2t}{7} = 1$$

$$t = \frac{7}{2}$$

3,5 jours

$$d) 0,25 P_0 = P_0 4^{t/7}$$

$$\frac{1}{4} = 4^{t/7}$$

$$\frac{t}{7} = -1$$

$$t = -7$$

1 semaine passée.

$$e) 8^{24} = 1 (4)^{t/7}$$

$$2^{72} = 2^{2t/7}$$

$$72 = \frac{2t}{7}$$

$$t = \frac{72 \times 7}{2}$$

$$t = 252 \text{ jours.}$$

30. $F(x) = 3^x$

$$\begin{aligned} F(x+2) - F(x) &= 3^{x+2} - 3^x \\ &= 3^x \cdot 3^2 - 3^x \\ &= 3^x(3^2 - 1) \\ &= 3^x(8) \\ &= 8F(x) \end{aligned}$$

32.

$$\frac{16^{x+2y}}{8^{x-y}} = 32$$

$$\frac{32^{x+3y}}{16^{x+2y}} = \frac{1}{8}$$

$$\frac{2^{4x+8y}}{2^{3x-3y}} = 2^5$$

$$\frac{2^{5x+15y}}{2^{4x+8y}} = 2^{-3}$$

$$2^{x+11y} = 2^5$$

$$2^{x+7y} = 2^{-3}$$

$$\begin{aligned} x + 11y &= 5 \\ (x + 7y = -3) & \quad x - 1 \end{aligned}$$

$$\begin{aligned} x + 11y &= 5 \\ -x - 7y &= 3 \end{aligned}$$

$$4y = 8$$

$$\boxed{y = 2}$$

$$x = 5 - 11(2)$$

$$\boxed{x = -17}$$

$$34 a) 9^{2x} = 2(9^x) + 3$$

$$(9^x)^2 = 2(9^x) + 3$$

$$\text{Posons } y = 9^x$$

$$y^2 = 2y + 3$$

$$y^2 - 2y - 3 = 0$$

$$(y-3)(y+1) = 0$$

$$y = 3 \quad y = -1$$

$$9^x = 3 \quad 9^x = -1$$

$$3^{2x} = 3^1$$

Aucune solution.

$$x = 1/2$$

$\{1/2\}$

$$\hookrightarrow 3(3^x) + 9(3^{-x}) = 28$$

$$3(3^x) + 9\left(\frac{1}{3^x}\right) = 28 \quad \text{Posons } 3^x = y$$

$$\left(3y + 9\left(\frac{1}{y}\right) = 28\right)^{\times y}$$

$$3y^2 + 9 = 28y$$

$$3y^2 - 28y + 9 = 0$$

$$3y^2 - 27y - y + 9 = 0$$

$$3y(y-9) - 1(y-9) = 0$$

$$(y-9)(3y-1) = 0$$

$$y = 9 \quad y = 1/3$$

$$3^x = 1/3$$
$$x = -1$$

$$3^x = 9$$
$$x = 2$$

$\{-1, 2\}$